Center for Academic Resources in Engineering (CARE)
Peer Exam Review Session

Math 241 – Calculus III

Midterm 1 Worksheet

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Feb 28, 5-6:30pm Nidhi and Danny Session 2: Mar 1, 7-8:30pm Pieter and David

Can't make it to a session? Here's our schedule by course:

https://care.engineering.illinois.edu/tutoring-resources/tutoring-schedule-by-course/

Solutions will be available on our website after the last review session that we host, as well as posted in the zoom chat 30 minutes prior to the end of the session

Step-by-step login for exam review session:

1. Log into Queue @ Illinois
2. Click "New Question"
3. Add your NetID and Name
4. Press "Add to Queue"
5. Join the zoom link in the staff message

Please do not log into the zoom call without adding yourself to the queue

Good luck with your exam!

1. Find $\vec{u} \times \vec{v}$ if $\vec{u} = \langle 3, -4, 1 \rangle$ and $\vec{v} = \langle 5, 2, -6 \rangle$

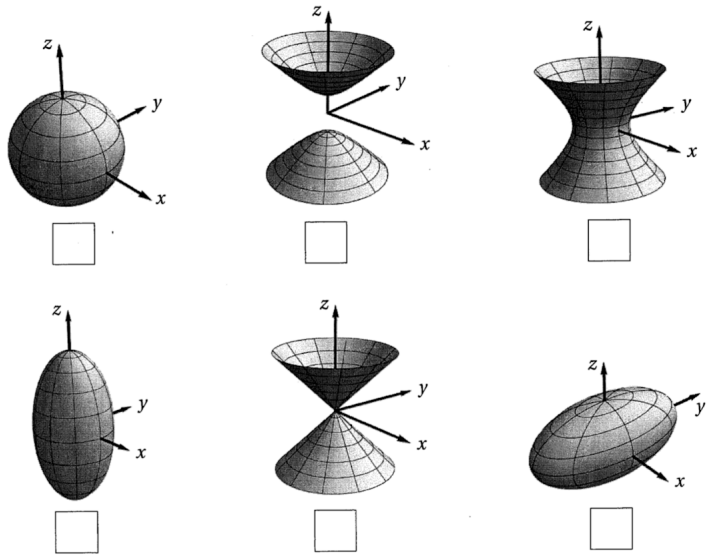
2. Find an equation for the plane that passes through the point $P = (1, 2, 3)$ and contains the line L given by the parametric equation:

$$x(t) = 1 - 3t \qquad y(t) = 3 \qquad z(t) = 6 + 2t$$

for $-\infty < t < \infty$

3. For each equation below, write the corresponding letter in the box next to the picture of the surface it describes

(A) $x^2 + y^2 + z^2 + 1 = 0$ (B) $4x^2 + y^2 + 4z^2 - 1 = 0$



4. Consider a function $f(t) = f(x(t), y(t), z(t)) = xyz - z^2$, where $x(t)$, $y(t)$, $z(t)$ are defined as followed:

$$x(t) = 2t^2 + 1 \qquad y(t) = 3 - \frac{1}{t} \qquad z(t) = 3$$

Find the following values:

(a) $f_z(3,1,2)$

(b) $\frac{d}{dt}x(t=0)$

(c) $\frac{d}{dt}f(t=1)$

5. Consider two points A and B.

$$A = (0,\ 7,\ 2)$$
$$B = (1,\ 2,\ 0)$$

(a) Find the vector that represents the displacement between the points (vector drawn from A to B)

(b) What is the projection of this vector onto $\vec{r} = \langle 5, -2, 7 \rangle$?

(c) What is the projection of the vector from part (a) onto the plane represented by the equation $5x - 2y + 7z = 10$

6. Consider the limit

$$\lim_{(x,y)\to(0,0)} \frac{y^4 \cos^2 x}{x^4 + y^4}$$

Does this limit exist? If so, what is its value? Justify your answer.

7. Determine if the two lines intersect. If so, find their point of intersection.

$$\begin{cases} x = 3t - 3 \\ y = -2t + 1 \\ z = 4t - 2 \end{cases} \qquad \begin{cases} x = 2s + 3 \\ y = 2s - 1 \\ z = s + 2 \end{cases}$$

8. Find the equation of the line of intersection of the following two planes: $2x + y - 2z = 2$ and $-2x + y + z = 6$.

9. Explain why $\vec{a} \cdot (\vec{a} \times \vec{b}) = 0$.

10. Describe in words what the contour lines of a function $f(x, y)$ would look like at a point (a, b) if, at that point, $\frac{\partial f}{\partial x} > 0$ and $\frac{\partial^2 f}{\partial x^2} < 0$