Center for Academic Resources in Engineering (CARE)
Peer Exam Review Session

Math 231 – Calculus II

Midterm 2 Worksheet Solutions

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Feb 20, 4-5:30pm Charlie and Greta Session 2: Feb 21, 4-5:30pm Alberto and Ayush

Can't make it to a session? Here's our schedule by course:

`https://care.engineering.illinois.edu/tutoring-resources/tutoring-schedule-by-course/`

Solutions will be available on our website after the last review session that we host, as well as posted in the Zoom chat 30 minutes prior to the end of the session

Step-by-step login for exam review session:

1. Log into Queue @ Illinois
2. Click "New Question"
3. Add your NetID and Name
4. Press "Add to Queue"
5. Join the Zoom link in the staff message

Please do not log into the Zoom call without adding yourself to the Queue

Good luck with your exam!

1. Evaluate the following Limit, if it exists:

$$\lim_{x\to\infty} \frac{x+7}{\sqrt{2x^2+7}}$$

$$= \lim_{x\to\infty} \frac{x(1+\frac{7}{x})}{\sqrt{x^2(2+\frac{7}{x^2})}}$$

$$= \lim_{x\to\infty} \frac{x(1+\frac{7}{x})}{x\sqrt{2+\frac{7}{x^2}}}$$

$$= \lim_{x\to\infty} \frac{1+\frac{7}{x}}{\sqrt{2+\frac{7}{x^2}}}$$

$$= \frac{1}{\sqrt{2}}$$

Note: If we were finding this limit as $x \to -\infty$, we would need to be cautious when removing x^2 from the square root. Here, since $x \to \infty \implies x > 0$, we can say $\sqrt{x^2} = x$.

2. Evaluate the following Limit, if it exists:

$$\lim_{x\to -5} \frac{x^2-25}{x^2+2x-15}$$

Method 1 (L'Hospital's Rule):

$$= \lim_{x\to -5} \frac{0}{0}$$

$$\implies \text{Apply L'Hospital's}$$

$$= \lim_{x\to -5} \frac{2x}{2x+2}$$

$$= \frac{-10}{-8} = \frac{5}{4}$$

Method 2 (factoring):

$$= \lim_{x\to -5} \frac{(x-5)(x+5)}{(x-3)(x+5)}$$

$$= \lim_{x\to -5} \frac{(x-5)}{(x-3)}$$

$$= \frac{-10}{-8} = \frac{5}{4}$$

3. Evaluate the following Limit, if it exists:

$$\lim_{x\to 0} x^8 \sin\left(\frac{\pi}{2x}\right)$$

$$-1 \le \sin\left(\frac{\pi}{2x}\right) \le 1$$

$$-x^8 \le x^8 \sin\left(\frac{\pi}{2x}\right) \le x^8$$

$$\lim_{x\to 0}(-x^8) = \lim_{x\to 0} x^8 = 0$$

By the Squeeze Theorem, we must have

$$\lim_{x\to 0} x^8 \sin\left(\frac{\pi}{2x}\right) = 0$$

4. Evaluate the following Integral:

$$\int_{1/3}^{1} \ln(3x)dx$$

$u = \ln(3x)$**,** $du = \frac{3}{3x}dx$**,** $dv = dx$**, and** $v = x$**.**

$\int udv = uv - \int vdu$

$= \ln(3x)x|_{\frac{1}{3}}^{1} - \int_{\frac{1}{3}}^{1} \frac{3}{3x}dx$

$= \ln(3x)x|_{\frac{1}{3}}^{1} - x|_{\frac{1}{3}}^{1}$

$= \ln(3) - \ln(0) - (1 - \frac{1}{3})$

$= \ln(3) - \frac{2}{3}$

5. Evaluate the following Integral:

$$\int_{0}^{\pi/4} \tan^5(x)\sec^4(x)dx$$

$u = \tan(x)$**,** $du = \sec^2(x)dx$**.**

$= \int_0^1 u^5 \sec^2(x)dx$

$= \int_0^1 u^5(1 + \tan^2(x))du$

$= \int_0^1 u^5(1+u^2)du$

$= \int_0^1 u^5 du + \int_0^1 u^7 du$

$= \int_0^{\frac{\pi}{4}} \tan^5(x)dx + \int_0^{\frac{\pi}{4}} \tan^7(x)dx$

$= \frac{1}{6}\tan^6(x)|_0^{\frac{\pi}{4}} + \frac{1}{8}\tan^8(x)|_0^{\frac{\pi}{4}}$

$= \frac{1}{6} + \frac{1}{8} = \frac{7}{24}$

6. Evaluate the following Integral:

$$\int \frac{1}{\sqrt{25+x^2}}dx$$

$x = 5\tan(\theta)$, $dx = 5\sec^2(\theta)$.

$= \int \frac{5\sec^2(\theta)d\theta}{\sqrt{25+5\tan(\theta)}}$

$= \int \frac{5\sec^2(\theta)d\theta}{\sqrt{25\sec^2(\theta)}}$

$= \int \sec(\theta)d\theta$

$= \ln|\sec(\theta) + \tan(\theta)| + C$

$= \ln|\frac{\sqrt{x^2+25)}}{5} + \frac{x}{5}| + C$

7. Evaluate the following Integral:

$$\int_0^4 \frac{1}{(x^2+16)^{\frac{3}{2}}}dx$$

$x = 4\tan(\theta)$, $dx = 4\sec^2(\theta)$.

$= \int_0^4 \frac{4\sec^2(\theta)d\theta}{4^3\sec^3(\theta)}$

$= \frac{1}{16}\int_0^4 \frac{1}{\sec(\theta)}d\theta$

$= \frac{1}{16}\int_0^4 \cos(\theta)d\theta$

$= \frac{1}{16}\sin\theta|_0^4$

$= \frac{1}{16}\frac{x}{\sqrt{x^2+16}}|_0^4 = \frac{1}{16}(\frac{4}{\sqrt{32}} - 0) = \frac{1}{16\sqrt{2}}$

8. Evaluate the following Integral:

$$\int \frac{2x^2 - x + 4}{x^3 + 4x} dx$$

$x^3 + 4x = x(x^2 + 4x)$

$\frac{2x^2-x+4}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$

$2x^2 - x + 4 = A(x^2 + 4) + (Bx + C)x$

$2x^2 - x + 4 = (A + B)x^2 + Cx + 4A$

$A + B = 2$, $C = -1$, and $4A = 4$

$A = 1, B = 1, C = -1$

$\int \frac{2x^2-x+4}{x^3+4x} dx = \int \frac{1}{x} + \frac{x-1}{x^2+4} dx$

$\int \frac{1}{x} + \frac{x}{x^2+4} - \frac{1}{x^2+4} dx = \ln|x| + \frac{1}{2}\ln(x^2 + 4) - \frac{1}{2}\tan^{-1}(\frac{x}{2}) + C$

For the second term, use U-sub where $u = x^2 + 4$. For the third term in the integral, use trig integral for tangent.

9. Evaluate the following Integral:

$$\int_0^{16} \frac{\sqrt{x}}{x+1} dx$$

Let $u = \sqrt{x}$, let $du = \frac{dx}{2\sqrt{x}} \to 2udu = dx$.

$= \int_0^4 \frac{2u^2 du}{u^2+1}$

$= 2\int_0^4 \frac{u^2+1-1}{u^2+1} du$

$= 2\int_0^4 1 - \frac{1}{u^2+1} du$

$= 2(u - \tan^{-1}(u))|_0^4 = 8 - 2\tan^{-1}(4)$

10. Evaluate the following Integral:

$$\int \arctan\left(\frac{2}{x}\right) dx$$

Let $u = \tan^{-1}(\frac{2}{x})$**,** $du = \frac{-2}{x^2}(\frac{1}{1+(\frac{2}{x})^2})dx$**,** $dv = dx$**, and** $v = x$**.**

$= \int \tan^{-1}(\frac{2}{x}) = x\tan^{-1}(\frac{2}{x}) + 2\int \frac{x}{x^2+4}dx$

Let $u = x^2 + 4$ **and** $du = 2xdx$**.**

$= \int \tan^{-1}(\frac{2}{x})dx = x\tan^{-1}(\frac{2}{x}) + \ln|x^2+4| + C$