



Center for Academic Resources in Engineering (CARE) Peer Exam Review Session

Phys 213 — University Physics: Thermal Physics

Final Worksheet Solutions

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: April 25, 8-10pm Jay and Greg Session 2: None

Can't make it to a session? Here's our schedule by course:

<https://care.engineering.illinois.edu/tutoring-resources/tutoring-schedule-by-course/>

Solutions will be available on our website after the last review session that we host, as well as posted in the zoom chat 30 minutes prior to the end of the session

Step-by-step login for exam review session:

1. Log into [Queue @ Illinois](#)
2. Click “New Question”
3. Add your NetID and Name
4. Press “Add to Queue”
5. Join the zoom link in the staff message

Please do not log into the zoom call without adding yourself to the queue

Good luck with your exam!

1. Which conditions are held constant in the following processes? (Refer to the formula sheet if you are not sure!)
 - a) Isothermal (constant temperature)
 - b) Isobaric (constant pressure)
 - c) Isochoric (constant volume)
 - d) Adiabatic (no heat added, no change in entropy)
2. What is the relationship between volume and pressure during isothermal and adiabatic processes for an ideal gas, respectively?

In an isothermal process, the temperature is constant and we can apply the ideal gas law: $pV = Nk_bT = \text{constant}$ (The ideal gas law is also used for isobaric and isochoric processes).

In an adiabatic process, $pV^\gamma = \text{constant}$ where $\gamma = \frac{C_p}{C_v} = (D + 2)/D$. Where C_v is the constant volume heat capacity, C_p is the constant pressure heat capacity, and D is the number of degrees of freedom. Since no heat is added (adiabatic) but work can still be done, $\Delta S = 0$. As the gas compresses, the temperature increases, and as the gas expands, the temperature decreases.

The following two questions refer to the setup described below.

A piston of volume 0.05 m^3 contains 5 moles of a monatomic ideal gas at 300 K. If it undergoes an isothermal process and expands until the internal pressure matches the external pressure, $P_E = 1 \text{ atm}$.

3. How much work is done by the gas on the environment?

- a) 7.42×10^3
- b) 1.13×10^4
- c) -1.13×10^4
- d) 1.83×10^4
- e) -1.83×10^4

The work done ON the gas is $dW_{on} = -pdV$ while work done BY the gas is $dW_{by} = pdV$. They are different by a minus sign, so make sure to check your signs. To find the total work, we integrate both sides of the equation, writing p in terms of V using the ideal gas law

$$W_{by} = \int_{V_i}^{V_f} p \, dV = \int_{V_i}^{V_f} \frac{Nk_bT}{V} \, dV = Nk_bT \ln\left(\frac{V_f}{V_i}\right)$$

The final volume can be found using the ideal gas law (make sure pressure is in Pascals)

$$V_f = \frac{Nk_bT}{P_E} = 0.123$$

4. Suppose that the piston undergoes an adiabatic expansion instead, what is the final volume of the piston, V_f ? (Values have units of cubic meters)

- a) 0.086
- b) 0.095
- c) 0.123

The P - V relation in an adiabatic process follows $pV^\gamma = \text{constant}$. Since the ideal gas is monatomic, $\gamma = \frac{5}{3}$. We can find the initial pressure using ideal gas law

$$P_i = \frac{Nk_bT}{V_i} = 249420 \text{ Pa}$$

which allows us to calculate the constant $PV^\gamma = 249420 \times 0.05^{\frac{5}{3}} = 1693$

After the expansion, we can use the $p - V$ relation again and plug in $P_f = P_E$ to get

$$P_E V_f^\gamma = 1693 \rightarrow V_f = 0.086$$

5. What is the difference between Helmholtz free energy and Gibbs free energy? In which situations would you use one over the other?

Helmholtz free energy is used when a constant volume system is in contact with a constant temperature environment. This is why Helmholtz free energy is used to calculate the available work done by a heated or cooled brick. The brick remains at constant volume as energy is transferred to or from a constant temperature environment.

On the other hand, Gibbs free energy is used when a constant pressure system is in contact with a constant temperature environment. This is more useful in chemical reactions where some sort of gaseous expansion causes the pressure of the system to remain constant.

6. When a system is colder than the temperature of the environment (i.e. $T_{sys} < T_{env}$) its free energy is:
- a) Smaller than its value when $T_{sys} = T_{env}$
 - b) Larger than its value when $T_{sys} = T_{env}$
 - c) The same as its value when $T_{sys} = T_{env}$

When the system reaches thermal equilibrium ($T_{sys} = T_{env}$), the free energy is minimized. So when the system has a different temperature from the environment, the free energy will always be larger than the minimum at equilibrium.

7. The second law of thermodynamics states that the change in entropy is always greater than or equal to zero, yet sometimes in our calculations we can get a negative entropy for a piece of our system. What does a negative entropy represent? And how do negative entropies not violate the second law?

A negative entropy means that the object we are looking at is becoming more ordered (i.e. gas cooling down and colliding less, smaller volume, etc...).

The second law states that the change in entropy *for the entire system considered* must be greater than or equal to zero, thus we can have individual components of a larger system that decrease in entropy, but that would require another part of the system gaining at least that much entropy.

8. Using the second law of thermodynamics, show that it is impossible for a heat engine to operate at $\epsilon = 1$.

Start with the fact that $\Delta S_{total} = \Delta S_H + \Delta S_C$ (the total change in entropy is the sum of the change in entropy of the hot and cold reservoir).

We know from the definition of temperature that $\frac{1}{T} = \frac{dS}{dU}$. This let's us rewrite the above as

$$\Delta S_{total} = \frac{\Delta U_H}{T_H} + \frac{\Delta U_C}{T_C} = -\frac{Q_H}{T_H} + \frac{Q_C}{T_C}$$

Where the last step is a result of $Q = \Delta U$ since the reservoirs do no work. Now using the second law, $\Delta S_{total} \geq 0$, we can see $\frac{Q_C}{Q_H} \geq \frac{T_C}{T_H}$

Combining this with the definition of engine efficiency: $\epsilon = \frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H} = 1 - \frac{Q_C}{Q_H}$

We can see the that there is a limit on ϵ

$$\epsilon \leq 1 - \frac{T_C}{T_H}$$

9. A Carnot heat engine (one operating at maximum efficiency) is operating between two reservoirs at T_H and T_C . Suppose that 1 kJ of heat must be added to the engine in order to produce 500 J of work and you measured the temperature of the cold reservoir to be 300 K, what is the temperature of the hot reservoir?

- a) 150 K
b) 200 k
c) 300 K
d) 450 K
e) 600K

The efficiency of this particular engine is $\epsilon = \frac{W}{Q_H} = \frac{500 \text{ J}}{1 \text{ kJ}} = \frac{1}{2}$. However, we are told that this is a Carnot engine, meaning it is operating at its maximum possible efficiency, which is given by

$$\epsilon_{max} = 1 - \frac{T_C}{T_H}$$

Knowing that the observed efficiency is the maximum efficiency (by definition of a Carnot engine), we can equate

$$\epsilon = \frac{1}{2} = 1 - \frac{T_C}{T_H}$$

Solving for the unknown value of T_H , we find the boxed answer choice.

10. Given an internal energy function $U(N) = \sin(N^2) + \ln(\alpha N)$ and an entropy function $S(N) = Ne^N$ determine the chemical potential as a function of N (T is constant).

Remember that the definition of chemical potential is the derivative of F with respect to N

$$\begin{aligned}\mu &= \frac{dF}{dN} = \frac{d}{dN}(U - TS) \\ &= \frac{d}{dN}[\sin(N^2) + \ln(\alpha N) - TNe^N] \\ &= 2N \cos(N^2) + \frac{1}{N} - T(e^N + Ne^N)\end{aligned}$$

11. Show that $-T\left(\frac{dS}{dN}\right)_{U,V} = \left(\frac{dF}{dN}\right)_{T,V}$ using the fundamental relation.

The fundamental relation of thermodynamics is

$$dS = \frac{1}{T}dU + \frac{p}{T}dV - \frac{\mu}{T}dN$$

Taking the first derivative, $\frac{dS}{dN}$, assuming constant U and V (meaning $dU = dV = 0$) and multiplying by $-T$, we obtain μ . Likewise, the second derivative, $\frac{dF}{dN}$, is the definition of chemical potential, which is μ . Thus the two expressions are equal.

12. A heat pump uses 200 J of work to remove 300 J of heat from a cold reservoir. How much heat would be delivered to the hot reservoir?

The relationship between the work done, the energy extracted from the cold reservoir, and the energy added to the hot reservoir is

$$Q_H = W + Q_C$$

This is also the equation used for a heat engine when all the energy originates from the hot reservoir. You can see this is true from conservation of energy.

Using this fact, and remembering that all these quantities are defined to be positive, we get

$$\boxed{Q_H = 500 \text{ J}}$$

13. Describe the difference between chemical equilibrium and thermal equilibrium.

Chemical equilibrium implies two systems have the same value of μ , while thermal equilibrium implies two systems have the same value of T . In the former, no more particles are exchanged, while in the latter, no more heat is exchanged.

14. Why does adding salt on the sidewalks in winter prevent ice from forming?

When we add salt to a water-based solution, the entropy of the solution increases because there are more microstates associated with it. This increased entropy makes it harder for the substance to freeze, since nature favors higher entropy systems. That is, the temperature would have to go even lower than it currently is for nature to favor the solid form of this substance over its liquid form. As a result, the freezing point decreases.

15. How does the phase diagram for water differ from other pure substances?

The phase diagram of water is unique in that as we increase pressure, the substance will become a liquid as opposed to remaining a solid. This indicates that the density of the solid form of water (ice) is less than the density of the liquid form, which is why ice floats in water.

