Center for Academic Resources in Engineering (CARE)
Peer Exam Review Session

Math 285 – Intro Differential Equations

Midterm 1 Worksheet

The problems in this review are designed to help prepare you for your upcoming exam. Questions pertain to material covered in the course and are intended to reflect the topics likely to appear in the exam. Keep in mind that this worksheet was created by CARE tutors, and while it is thorough, it is not comprehensive. In addition to exam review sessions, CARE also hosts regularly scheduled tutoring hours.

Tutors are available to answer questions, review problems, and help you feel prepared for your exam during these times:

Session 1: Feb 24, 7-8:30pm Elena and Amy

Session 2: March 1, 7-8:30pm Rehnuma and Chris

Can't make it to a session? Here's our schedule by course:

https://care.engineering.illinois.edu/tutoring-resources/tutoring-schedule-by-course/

Solutions will be available on our website after the last review session that we host, as well as posted in the zoom chat 30 minutes prior to the end of the session

Step-by-step login for exam review session:

1. Log into Queue @ Illinois
2. Click "New Question"
3. Add your NetID and Name
4. Press "Add to Queue"
5. Join the zoom link in the staff message

Please do not log into the zoom call without adding yourself to the queue

Good luck with your exam!

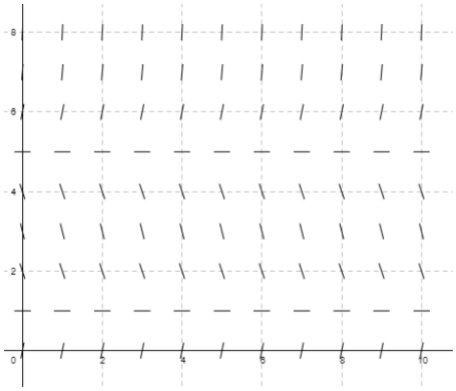
1. Consider the initial value problem:

$$(16 - t^2)y' + t^3 y = \cos\left(\frac{t}{2}\right) \qquad y(1) = -5$$

On what interval is the unique solution certain to exist?

A) (0,2π)
B) (-2π,0)
C) (-4,0)
D) (-4,4)
E) (-∞,-4)

2. Which equation produces the direction field below?



A) $\frac{dy}{dx} = (x-1)(x-5)$
B) $\frac{dy}{dx} = xy$
C) $\frac{dy}{dx} = (y-1)(y-5)$
D) $\frac{dy}{dx} = (x-1)(y-5)$
E) $\frac{dy}{dx} = y^2$

3. Which of the following equations are linear?

(I) $\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + e^y = 6t + 5$

(II) $(2t^3 + 6)\frac{\mathrm{d}^5y}{\mathrm{d}t^5} - \frac{\mathrm{d}^3y}{\mathrm{d}t^3} + 4y = t\cos(t-1)$

(III) $u_y = uu_{xx} - u_{xy}$

(IV) $u_{xx} + xu_{xt} + t^2u_{tt} = \sin(x + 2t)$

A) (I), (II), (III)
B) (II) and (IV)
C) (I) and (IV)
D) (II)
E) (IV)

4. What's the order of the following differential equation?

$$\cot(y)y''' + (t^2 + t + 9)y' - \ln(xy^2)y + 6y^9 = \sin(3t^5 + 1)$$

A) 9
B) 1
C) 3
D) 5
E) None; not a linear ODE

5. Consider the functions:

$$f(x,t) = e^{-4t}\sin(x) \qquad g(x,t) = e^{-t}\cos(2x) \qquad h(x,t) = \sin(x)\cos(t)$$

Which of these functions are solutions of the partial differential equation $u_t = 4u_{xx}$

A) f and g
B) f and h
C) g only
D) g and h
E) f only

6. Consider the population model. Constants $T, r > 0$.

$$y' = -r(1 - \frac{y}{T})y$$

Find and define the equilibrium solutions

A) $y(t) = 0$ is asymptotically stable; $y(t) = T$ is unstable
B) $y(t) = r$ is asymptotically unstable; $y(t) = T$ is stable
C) $y(t) = r$ is asymptotically stable; $y(t) = 0$ is unstable
D) $y(t) = 0$ is asymptotically unstable; $y(t) = T$ is stable
E) None of the above

7. What's the solution to

$$\frac{dz}{dx} = -4z - 3$$

A) $z = Ce^{-4x} + \frac{3}{4}$
B) $z = Ce^{4x} + \frac{3}{4}$
C) $z = Ce^{-4x} - \frac{3}{4}$
D) $z = Ce^{-3x} - \frac{4}{3}$
E) $z = Ce^{-4x} - \frac{4}{3}$

8. Solve

$$\frac{dy}{dx} - \frac{2y}{7} = \frac{3e^{5x}}{y^6}$$

(Hint: Bernoulli)

9. Solve

$$x\left(\frac{dy}{dx}\right) = 2y + x$$

10. Consider $y'' + 5y' + 6y = 0$ with $y(0) = 1$ and $y'(0) = -2$. Find the fundamental set of solutions (and prove that it is). (Hint: Prove with Wronskian)